Elasticity - Elastic, plastic materials, stress, strain and their units and young's modulus Exercise 2.5.10

Elastic material

The Elastic materials are those materials that have the ability to resist a distorting or deforming influence or force, and then return to their original shape and size when the same force is removed.

Linear elasticity is widely used in the design and analysis of structures such as beams, plates and sheets.

Elastic materials are of great importance to society since many of them are used to make clothes, tires, automotive spare parts, etc.

Characteristics of elastic materials

When an elastic material is deformed with an external force, it experiences an internal resistance to the deformation and restores it to its original state if the external force is no longer applied.

To a certain extent, most solid materials exhibit elastic behavior, but there is a limit of the magnitude of the force and the accompanying deformation within this elastic recovery.

A material is considered as elastic if it can be stretched up to 300% of its original length.

For this reason there is an elastic limit, which is the greatest force or tension per unit area of a solid material that can withstand permanent deformation.

For these materials, the elasticity limit marks the end of their elastic behavior and the beginning of their plastic behavior. For weaker materials, the stress or stress on its elasticity limit results in its fracture.

The elasticity limit depends on the type of solid considered. For example, a metal bar can be extended elastically up to 1% of its original length.

However, fragments of certain gummy materials may undergo extensions of up to 1000%. The elastic properties of most solid intentions tend to fall between these two extremes.

Maybe you might be interested How to Synthesize an Elastolic Material?

Examples of elastic materials

1 Natural gum
2 Spandex or lycra
3 Butyl Rubber (GDP)
4 Fluoroelastomer
5 Elastomers
6 Ethylene-propylene rubber (EPR)
7 Resilin
8 Styrene-butadiene rubber (SBR)
9 Chloroprene
10 Elastin
11 Rubber Epichlorohydrin
12 Nylon
13 Terpene
14 Isoprene Rubber
15 Poilbutadiene
16 Nitrile Rubber
17 Vinyl stretch
18 Thermoplastic elastomer
19 Silicone rubber
20 Ethylene-propylene-diene rubber (EPDM)
21 Ethylvinylacetate (EVA or foamy gum)
22 Halogenated butyl rubbers (CIIR, BIIR)
23 Neoprene

Plastic Material

Plastic Material Classification

"Plastic material" is a term that refers to a large class of polymers, separated into various groups and sub-groups. Before starting the chapter on the uses and subsequent recycling of plastic, let us establish a general classification of these thermosetting resins or thermo-plastics (the two big groups into which we include elastomers) by detailing their properties, their make-up, their aspect, and their final uses, while explaining which ones are recyclable.

Thermoplastics

Remember that thermoplastic is a material whose structure and viscosity can be modified both ways through heating or cooling. This large family of materials is commonly used by many industries and is easily integrated into France's recycling cycles.

The following polymers are some examples of plastic material:

1 Polyolefins
2 Vinyl polymers
3 Polystyrenes
4 Acrylate and methacrymate polymers
5 Polyamide
6 Polycarbonates
7 Celluloid
8 Linear polyesters
9 Polyfluorethane
10 Polyacetal
11 Polysulfone
12 Polyphenylene sulfide
13 Modified polyphenylene oxide (PPO)

Thermosetting plastic

Thermosetting plastic is a compound that, during condensation polymerisation (and/or implementation), when submitted to a catalyst or a temperature increase, irreversibly cures. the structure, shape, or rigidity of the manufactured plastic object can not be modified again, and the material is rarely recycled.

This type of plastic includes the following types of compounds:

1 Unsaturated polyster

2 Phenol formaldehyde resins

3 Melamine resins

4 Polyepoxides

5 Polyimide

6 Polyurethane

7 Polyorganosiloxanes

Generally in any industry the material used are elastic in nature. Hence if a material is subjected to an external load, it undergoes deformation. During the deformation process the material will offer a resistance against the deformation. In case if the material fails to put up full resistance to the external load, the deformation continues until rupture takes place. Hence it is important to have a considerable knowledge about the materials and their properties for designing and fabricating.

Strain

When an external forces acting on a material, there is a change in its dimension and shape. The deformation is called strain. Thus, strain is the ratio between the change in dimension of a material to its original dimension. It has no unit. It is represented by e (Epsilon)

$$\text{Strain} = \frac{\text{Change in dimension } (\delta \ell)}{\text{Original dimension } (\ell)}$$

Linear or Longitudinal strain

It is the ratio between the change in length of the material to its original length.

$$\text{Linear Strain} = \frac{\text{Change in length } (\delta \ell)}{\text{Original length } (\ell)}$$

Lateral Strain

It is the ratio between change in cross sectional area of material to its original area.

$$\text{Lateral Strain} = \frac{\text{Change in area}}{\text{Original Area}}$$

Volumetric Strain

It is the ratio between change in volume of material to its original volume.

$$\text{Volumetric Strain} = \frac{\text{Change in volume}}{\text{Original Volume}}$$

Poisson's ratio

It is a ratio between lateral strain and linear strain.

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{1}{m}$$

Examples

1 Calculate the tensile strain when a force of 3.2 kN is applied to a bar of original length 280 cm extends the bar by 0.5 mm (Fig 1 & Fig 2)

Fig 1

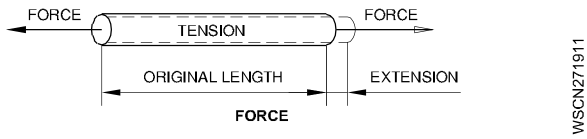
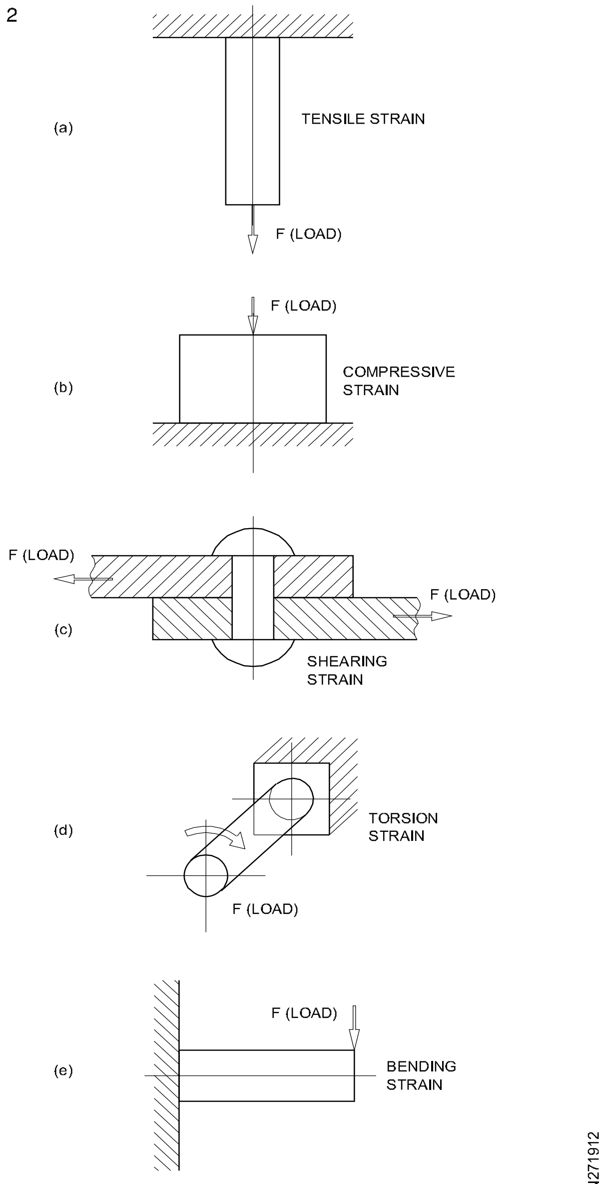


Fig 2



TYPES OF STRAIN

Force = 3.2 kN

Original length (L) = 280 cm

Increased length($\Delta \ell$) = 0.5 mm = 0.05 cm

$$\text{Tensile Strain} = \frac{\text{Increased length}}{\text{Original Length}}$$

$$= \frac{0.05}{280}$$

= 0.0001786

2 **A steel rod used for brake operation is 1.50 m long. When it is subjected to a tensile force the extension produced is 0.5 mm. Find the strain in the rod.**

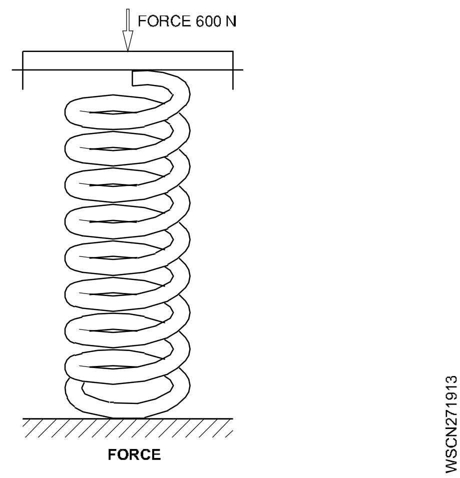
$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}}$$

$$= \frac{0.5}{1.5 \times 1000}\left(\frac{mm}{mm}\right)$$

Strain in the brake rod = 0.0003

3 **A helical spring is loaded with a force of 600 Newton and is compressed by 30mm. What would be the load required to compress it to 10 mm (Fig3)**

Fig 3



Solution

$$\text{Spring stiffness} = \frac{\text{Applied load}}{\text{Deflection}}$$

$$= \frac{600}{30}\left(\frac{N}{mm}\right) = 25\left(\frac{N}{mm}\right)$$

Load required to compress the spring by 10 mm

= spring stiffness x deflection

= 25 (N/mm) x 10(mm)

Load required = 250 N

4 **Helical spring is loaded with a force of 400 Newton and it is compressed by 18 mm. What would be the load required to compress it to 6 mm?**

Given force = 400 Newton

Deflection = 18 mm

Spring Stiffness = Force / Compressed length

= 400 /18

= 22.22 Newton / mm

Force required to compress the spring into 6 mm = Spring stiffness $\times$ Deflection

Load required = 22.22 $\times$ 6

= 133.32 N

5 **Calculate the tensile strain when a force of 3.2 KN is applied to a bar of original length 2.8 m extends the bar by 0.5 mm.**

Force F = 3.2 KN

Original length L = 280 cm

Increased length($\Delta\ell$) = 0.5 mm = 0.05 cm

Tensile strain = ?

$$\text{Strain} = \frac{\Delta\ell}{L}$$

$$= \frac{0.05}{280}$$

= 0.0001786

6 **A metal bar is 2m long. When 5.5 tonne is applied its length becomes 1.995 m. Find the compressive strain?**

Force F = 5.5 KN

Original length L_1 = 2 m

Final length L_2 = 1.995 m

Increased length($\Delta\ell$) = 2 - 1.995 = 0.005 m

$$\text{Compressive strain} = \frac{\Delta\ell}{L}$$

$$= \frac{0.005}{2}$$

= 0.0025

7 **When a steel rod of 4mm diameter experienced the load of 200 Kg. It is found to be elongated by 1.5 mm from the original length of 1500 mm. Calculate the strain.**

Force F = 200 Kg.

Original length L_1 = 1500 mm

$\Delta\ell$ = 1.5 mm

Strain = ?

$$\text{Compressive strain} = \frac{\Delta\ell}{L}$$

$$= \frac{0.005}{2}$$

= 0.001

8 **An iron rod of length 1 metre and 1 cm diameter gets elongated by 1 cm. When a force of 100 Kg is applied at one end. Calculate the strain developed in the rod.**

Force F = 100 Kg.

Original length L_1 = 1 m = 100 cm

$\Delta \ell$ = 1 cm

Strain = ?

$$\text{Compressive strain} = \frac{\Delta \ell}{L}$$

$$= \frac{1}{100}$$

$$\mathbf{= 0.01}$$

Stress

The internal opposite force to the external load per unit area is known as stress. The unit of stress depends upon the force applied and area of original cross-section of material. It is represented by σ (Sigma)

$$\therefore \text{Stress} = \frac{\text{Forced applied}}{\text{Area of original cross section}}$$

$$= \frac{\text{Load (or) Force}}{\text{Area}} \left(\frac{N}{mm^2} \text{ or } \frac{Kg}{cm^2} \right)$$

$$\text{Shear stress } (\tau) = \frac{F}{A} \left(\frac{N}{cm^2} \text{ or } \frac{Kg}{cm^2} \right)$$

Types of Stress

1 Tensile stress

2 Compressive stress

3 Shear stress

4 Torsional Stress

1 **Tensile stress:** When a material is subjected to two equal and opposite axial pulls, the material tends to increase in length. The resistance offered against this increase in length is called tensile stress. The corresponding strain is called tensile strain. (Fig 4)

Fig 4

E.g.:

1 When brake is applied the brake rod is under tensile stress.

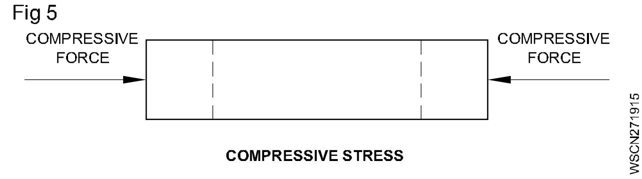
2 During tightening of bolt or nut.

3 Belt driving the fan.

4 Crane rope (When rope is pulling)

2 **Compressive stress:** When a material is subjected to two equal and opposite axial pushes, the material tends to decrease in length. The resistance offered against the decrease in length is called compressive stress. The corresponding strain is called compressive strain. (Fig 5)

Fig 5



$$\text{Compressive stress} = \frac{\text{Axial push}}{\text{Area of cross section}}$$

$$\text{Compressive stress} = \frac{\text{Decrease in length}}{\text{Original length}}$$

Eg.

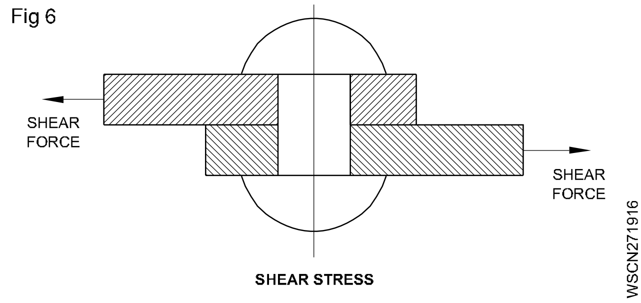
1 Compressive stress on connecting rod on the first part of power stroke

2 Compressive stress on push rod during valve opening

3 Clutch lining when the clutch is engaged

3 **Shear stress:** When a material is subjected to two equal and opposite forces acting tangentially across the resisting section, the body tends to be sheared off across the cross section. The stress included is called shear stress. It is represented by τ. The corresponding strain is called shear strain. (Fig 6)

Fig 6



$$\text{Shear stress } (\tau) = \frac{F}{A} \left(\frac{N}{cm^2} \text{ or } \frac{Kg}{cm^2} \right)$$

Eg.

1 Rivets

2 Gudgeon Pin

3 Spring shackle pin

4 Brake rod rivets

5 Chassis rivets

6 Fly wheel holding bolts

7 Swivel pins

8 Gear box shaft

9 Axle shaft

4 **Torsional stress:** When a shaft is subjected to the action of two equal and opposite couples acting in parallel planes, then the shaft is said to in torsion. The stress set up by the torsion is known as torsional shear stress.

Eg.

1 Rear axle

2 Crank shaft

3 Coil springs

4 Propeller shaft

5 Starter motor armature shaft

Examples

1 **A steel wire 3 mm dia. is loaded in tension with a weight of 50 kg. Find out the stress developed.**

Diameter of the steel wire = 3 mm

Radius = 1.5 mm

Weight = 50 kg

Stress $= \frac{\text{Force(F)}}{\text{Area(A)}}$

Area of circular wire (A) $= \pi r^2$

$= \frac{22}{7} \times 1.5 \times 1.5$

$= \frac{49.5}{7} = 7.07 \text{mm}^2$

Stress $= \frac{50}{7.07}$

= 7.072 Kg/mm²

2 **A force of 500 N is applied on a metallic wire of 5mm diameter. Find the stress.**

Diameter of the wire = 5 mm

Radius = 2.5 mm

Force = 500 Newton

Stress $= \frac{\text{Force(F)}}{\text{Area(A)}}$

Area of circular wire (A) $= \pi r^2$

$= \frac{22}{7} \times 2.5 \times 2.5$

$= \frac{137.5}{7} = 19.64 \text{mm}^2$

Stress $= \frac{500}{19.64}$

= 25.46 N/mm²

3 **A load of 600 kg is placed on a hollow cast iron cylinder of 200 mm outer diameter and 100 mm internal diameter. Find the stress on the cylinder.**

Hollow cylinder

Outer diameter (D) = 200 mm = 20 cm

Outer radius (R) = 10 cm

Internal diameter (d) = 100 mm = 10 cm

Inner radius (r) = 5 cm

Weight = 600 kg

Stress $= \frac{\text{Force(F)}}{\text{Area(A)}}$

Area $= \pi (R + r)(R - r)$

$= \frac{22}{7} \times (10 + 5) \times (10 - 5)$

$= \frac{22}{7} \times 15 \times 5$

$= \frac{1650}{7} = 235.7 \text{cm}^2$

Stress $= \frac{600}{235.7}$ kg/cm²

= 2.546 kg/cm²

4 **Calculate the minimum cross section area of a M.S. bar to withstand a load 6720 kg. Take the maximum stress of the material as 698.2 kg/cm².**

Weight = 6720 kg

Maximum stress = 698.2 kg/cm²

Stress $= \frac{\text{Force(F)}}{\text{Area(A)}}$

Area(A) $= \frac{6720}{698.2}$

= 9.625 cm²

To calculate diameter

Area $= \frac{\pi d^2}{4}$

d^2 = 4 x 9.625 x $\frac{7}{22}$

$= \frac{134.75}{11}$

d^2 = 12.25

d = 3.5 cm

5 **A load of 300 kg hanging from a rod of 3 metre length and 5 mm diameter extends it by 4 mm. Find the stress in the material and the strain it causes.**

Length of the rod = 3 m = 3000 mm

Increased length = 4 mm

Diameter = 5 mm;

Radius = 2.5 mm

Weight = 300 kg

Strain $= \frac{\text{Change in length}}{\text{Original length}}$

$= \frac{4}{3000} = 0.00133$

Stress $= \frac{\text{Force(F)}}{\text{Area(A)}}$

Area of circular rod (A) $= \pi r^2$

$= \frac{22}{7} \times 2.5 \times 2.5$

$= \frac{137.5}{7}$

= 19.643 mm^2

Stress $= \frac{300}{19.643}$

= 15.273 kg/mm^2

6 **Find the force required to punch a hole of 10 mm dia. in a 1 mm thick plate, if the allowable shear stress is 50 N/mm^2.**

Thickness of the plate= 1 mm

Dia. of the punch = 10 mm

Shear stress = 50 Newton/mm^2

Force = Shear stress x area

Shear area = Circumference x thickness

$= \pi dt$

$= \frac{22}{7} \times 10 \times 1$

$= \frac{220}{7} = 31.43$ mm^2

Force = 50 x 31.43

= 1571.5 Newton

7 **A hole of 30 mm diameter is punched in a plate of 5 mm thickness. If the shear stress is 400 kg/cm^2. Find the force required to punch the hole.**

Thickness of the plate = 5 mm = 0.5 cm

Diameter of the punch = 30 mm = 3 cm

Shear stress = 400 kg/cm^2

Force = Shear stress x area

Shear area = Circumference x thickness

$= \pi Dt$

$= \frac{22}{7} \times 3 \times 0.5$

$= \frac{33}{7} = 4.71$ mm^2

Required force = 400 x 4.71

= 1885.71 kg

8 **What force will be required to shear off a bar of 30 mm dia. if the ultimate shear stress of the material is 35 kg/mm^2.**

Diameter of the bar = 30 mm

Shear stress = 35 kg/mm^2

Stress(σ) $= \frac{\text{Force(F)}}{\text{Area(A)}}$

$35 = \frac{F}{\pi \times 15 \times 15}$

$F = 35 \times \pi \times 15 \times 15$ kg

= 24750 kg

9 **A Hole of 2 cm dia is to be punched out of a plate of 1.4 cm thick. If the force applied to the punching die is 12 KN. Calculate the shear stress.**

Dia. of the hole = 2cm

Thickness = 1.4 cm

Force = 12 KN

Shear stress = ?

Punched out area = Circumference of the hole $\times$ Thickness

$= 2\pi r \times t$ unit^2

$= 2 \times \pi \times 1 \times 1.4$

$= 2.8\pi$ cm^2

Shear stress (τ) $= \frac{F}{A}$

$= \frac{12KN}{2.8\pi \text{ cm}^2}$

= 1.364 KN/cm^2

10 A square rod of 10 mm side is tested for a tensile load of 1016 kg. Calculate the tensile stress?

Side of square rod a = 10 mm

Tensile force F = 1016 kg

Tensile stress σ = ?

$$\text{Stress}(\sigma) = \frac{\text{Force(F)}}{\text{Area(A)}}$$

$$= \frac{\text{Force}}{a^2}$$

$$= \frac{1016}{10 \times 10}$$

$$\mathbf{= 10.16\ Kg/mm^2}$$

11 A M.S. tie bar 3.5 cm dia. is under a state of stress which carries a load of 6720 kg. Find the intensity of stress in the material.

d = 3.5 cm

r = 1.75 cm

F = 6720 kg

$$\text{Stress}(\sigma) = \frac{\text{Force(F)}}{\text{Area(A)}}$$

$$= \frac{\text{Force}}{\pi r^2}$$

$$= \frac{6720}{3.14 \times 1.75 \times 1.75}$$

$$= \frac{6720}{9.616}$$

$$\mathbf{= 698.8\ Kg/cm^2}$$

12 A rivet of 10 mm dia. is subjected to a double shear force of 1.5 KN. Find the shear stress in the rivet.

dia. of the rivet = 10 mm

r = 5 mm

Shear stress = ?

Double shear force is acting on the rivet, consider the area as double.

$$\text{Stress} = \frac{F}{2\text{Area}}$$

$$= \frac{1.5}{2 \times 3.14 \times 5 \times 5}$$

$$\mathbf{= 0.00955\ KN/mm^2}$$

Elasticity and Elastic limit

When an external force acts on a body, the body tends to under go some deformation. If the external force is removed and the body comes back to its original shape and size (Which means the deformation disappers completely). The body is known as elastic body. This property by virtue of which certains materials return back to their original position after the removal of the external force is called elasticity.

The body will regain its previous shape and size only when the deformation caused by the external force is with in a certain limit. Thus there is a limiting value of force up to and within which the deformation completely disapperas on the removal of the force. The value of stress corresponding to this limiting force is known as elastic limit of the material.

If the external force is so large that the stress exceeds the limit, the material loses to some extent its property of elasticity. If now the force is removed, the material will not return to its original shape and size and there will be a residual deformation in material.

Yield point

The yeild point of a material is the point at which there is a marked increase in elongation without increase in load.

Hooke's law

Robert Hooke discovered a relationship between stress and strain. According to Hooke's law stress is proportional to strain within elastic limit.

Young's Modulus or Modulus of Elasticity

The ratio of stress to strain within elastic limit is known as young's modulus or modulus of elasticity. This is expressed by a symbol "E". The unit of Young's modulus is same that of stress.

$$\therefore \quad \text{Young's modulus (E)} = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of Rigidity

The ratio of shear stress to shear strain is known as "modulus of rigidity" represented by symbol "N".

$$\therefore \quad \text{Modulus of Rigidity (N)} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

Bulk Modulus

When a body is subjected to three mutually perpendicular forces of the same intensity, the ratio of volumetric stress to the volumetric strain is known as Bulk Modulus. It is usually represented by the letter K.

$$\therefore \quad \text{Bulk Modulus (K)} = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

Relationship between three moduli for a given material

The relationship between three moduli for a given material is as follows :

$$E = 2N\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right).$$

Where

E = Young's modulus of elasticity

N = Modulus of rigidity

K = Bulk modulus

$\frac{1}{m}$ = Poisson's ratio

Example

1 A steel rod of 10 mm diameter and 175 mm long is subjected to a tensile load of 15 kN. If E = 2 x 10^5 N/mm², calculate the change in length.

Tensile load = 15 kN = 15000 N

Area of cross section $= (\pi r^2) = \frac{22}{7} \times 5 \times 5\ mm^2 = 78.57$

$\therefore$ Stress $= \frac{15000N}{0.785 \times 100mm^2} = 191\ N/mm^2$

Young's modulus E $= \frac{Stress}{Strain}$

$E = 2 \times 10^5\ N/mm^2 = \frac{191 N/mm^2}{Strain}$

$\therefore$ Strain $= \frac{191}{2 \times 10^5}$

Change in length $= \frac{175 \times 191}{2 \times 10^5} mm$

= 0.167 mm.

2 A bar of steel 2.5 cm diameter was subjected to compressive load of 4500 kg. The compression in a length of 20 cm was found to be 0.008 cm. Find the Young's modulus of elasticity of bar.

Solution

Diameter of bar d = 2.5 cm

Force applied i.e. compressive load = 4500 kg

Original length of bar = 20 cm

Change in length = 0.008 cm

$\therefore$ Area of original cross-section $= \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times 2.5^2$

$= \frac{\pi \times 6.25}{4} cm^2$

$\therefore$ Stress $= \frac{\text{Force applied}}{\text{Area of original cross section}}$

$= \frac{4500}{\frac{\pi \times 6.25}{4}}$

$= \frac{4500 \times 4}{\pi \times 6.25}$

$= \frac{2880}{\pi}$

$\therefore$ Stress $= \frac{2880}{\pi} Kg/cm^2$

$\therefore$ Strain $= \frac{\text{Change in length}}{\text{Original length}}$

$= \frac{0.008}{20} = \frac{8/1000}{20}$

$= \frac{8}{20 \times 1000} = \frac{4}{10000}$

$\therefore$ Strain $= \frac{4}{10000}$

$\therefore$ Young's modulus $= \frac{Stress}{Strain}$

$= \frac{2880}{\pi} \div \frac{4}{10000}$

$= \frac{2880}{\pi} \times \frac{10000}{4}$

$= \frac{7200000}{\pi}$

= 2292000 Kg/cm²

= 2.292 x 10^6 Kg/cm²

3 A force of 10 tonnes is applied axially on a rod of 12 cm dia. the original length is 100 mm.If modulus of elasticity is 2 x 10^{12} kg/cm². Calculate stress and strain developed in the rod.

Solution

Force applied = 10 tonnes = 10 x 1000 kg

= 10^4 kg

Diameter (d) = 12 mm = 1.2 cm

Young's modulus (E) = 2 x 10^{12} kg/cm²

$$\text{Stress} = \frac{\text{Force applied}}{\text{Area of original cross section}}$$

$$= \frac{10^4}{\frac{\pi}{4} \times \frac{12}{10} \times \frac{12}{10}}$$

$$= \frac{10^4 \times 4 \times 10 \times 10}{\pi \times 12 \times 12}$$

$$= \frac{10^6}{36\pi}$$

$= 8841$ kg/cm^2

∴ Stress $= 8841$ kg/cm^2

We know

$$\frac{\text{Stress}}{\text{Strain}} = \text{Young's modulus}$$

Strain x Young's modulus = Stress

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}}$$

$$= \frac{8841}{2 \times 10^{12}}$$

$$= \frac{4420.5}{10^{12}}$$

$= 4420.5 \times 10^{-12}$

Stress $= 8841$ kg/cm^2

Strain $= 4420.5 \times 10^{-12}$

4 A bar of 100 cm elongates to 101.36 cm when a load of 15000 kg is applied to it. Take the area of cross section of bar as 10 cm^2. Find the stress, strain and youngs modulus.

$L_1 = 100$ cm

$L_2 = 101.36$ cm

$\Delta\ell = L_2 - L_1$

$= 101.36 - 100 = 1.36$ cm

F $= 15000$ kg

A $= 10$ cm^2

$$\text{Stress} = \frac{\text{Force(F)}}{\text{Area(A)}}$$

$$= \frac{15000}{10}$$

$= 1500$ kg/cm^2

$$\text{Strain} = \frac{\Delta\ell}{L}$$

$$= \frac{1.36}{100}$$

$= 0.0136$

$$\text{Youngs modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$E = \frac{1500}{0.0136}$$

$= 110300$ kg/cm^2

5 What force is required to stretch a steel wire of 10 mm long and 10 mm dia. to double its length. E of steel is 205 KN/cm^2.

d $= 10$ mm $= 1$ cm

r $= 0.5$ cm

$L_1 = 1$ cm

$L_2 = 2$ cm

$\Delta\ell = L_2 - L_1 = 2 - 1 = 1$ cm

E $= 205$ KN/cm^2

$$\text{Strain} = \frac{\Delta\ell}{L} = \frac{1}{1} = 1$$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$205 = \frac{\text{Stress}}{1}$$

Stress $= 1 \times 205 = 205$ KN/cm^2

$$\text{Stress} = \frac{\text{Force(F)}}{\text{Area(A)}}$$

$$205 = \frac{\text{Force}}{3.14 \times 0.5 \times 0.5}$$

Force $= 205 \times 3.14 \times 0.5 \times 0.5$

$= 161$ KN

6 A wire of 1.6 cm diameter is subjected to a tensile load of 2000 Kg. Find the stress and strain if youngs modulus = 2×10^6 kg/cm^2.

F $= 2000$ kg

d $= 1.6$ cm

r $= 0.8$ cm

E $= 2 \times 10^6$ Kg/cm^2

$$\text{Stress} = \frac{F}{A}$$

$$= \frac{2000}{\pi r^2}$$

$$= \frac{2000}{3.14 \times 0.8 \times 0.8}$$

$$= \frac{2000}{2.0096}$$

$= 995.2\ kg/cm^2$

$$\text{Youngs modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$2 \times 10^6 = \frac{995.2}{\text{Strain}}$$

$$\text{Strain} = \frac{995.2}{2 \times 10^6}$$

= 0.0005

7 **A tensile load of 2000 kg is applied on a rectangular rod of 2 cm x 1 cm whose length is 2 metres. Calculate the elongation in length as E = 2 x 10⁶ Kg/cm².**

F = 2000 Kg.

L_1 = 2 m = 200 cm

E = $2 \times 10^6\ kg/cm^2$

Rectangular rod length = 2 cm

Breadth = 1 cm

$$\text{Stress}(\sigma) = \frac{\text{Force(F)}}{\text{Area(A)}} = \frac{\text{Force}}{l \times b}$$

$$= \frac{2000}{2 \times 1}$$

$= 1000\ kg/cm^2$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$2 \times 10^6 = \frac{1000}{\text{Strain}}$$

$$\text{Strain} = \frac{1000}{2 \times 10^6}$$

= 0.0005

$$\frac{\Delta \ell}{L_1} = \text{Strain}$$

$$\frac{\Delta \ell}{200} = 0.0005$$

$\Delta \ell$ = 200 x 0.0005

= 0.1 cm

∴ **Elongated length = 0.1 cm**

Assignment

Strain

1 Find the compressive strain if a metal bar is 150 cm long. When 2.5 KN is applied, its length becomes 148.6 cm.

2 Calculate the strain if a metallic bar is 150 cm long. When 2500 kg is applied its length becomes 150.5 cm.

3 Find the strain it causes if a load of 300 kg hanging from a rod of 3 metres length and 5 mm diameter extends it by 4 mm.

4 A tensile force of 10 kg is applied on a copper wire of diameter 1 cm. So that the length of wire increases by 5 mm. If the original length of wire was 2 metres, findout the strain.

Stress

1 Calculate the intensity of stress in the material if a copper rod of 40 mm diameter is subjected by tensile load of 4000 Newtons.

2 Calculate the intensity of stress if a mild steel rod having a cross sectional area of 40 mm² is subjected to the load of 1000 kg.

3 Calculate the tensile stress if a square rod of 10 mm side is tested for a tensile load of 1000 kg.

4 Calculate the maximum stress if a bar of 9 cm² cross sectional area 300 cm long carries a tensile load of 3500 kg.

5 Find out the stress on the rod. if a load of 500 kg is placed on a M.S.rod of dia. 35 mm.

6 A metallic bar of 8 cm diameter is under stress carrying a load of 8620 N. Calculate the intensity of stress.

7 A steel wire 2 mm diameter is loaded in tension with a weight of 20 kg. Find out the stress developed.

8 A rod having a cross sectional area of 25 mm² is subjected to a load of 1500 kg. Find out stress on the rod.

9 A square rod of 10 mm side is tested for a tensile load of 2500 kg. Calculate the tensile stress of the rod.

Youngs modulus

1 A piece of wire 2 m long, 0.8 mm^2 in cross section increases its length by 1.6 mm on suspension of 8 kg weight from it. Calculate the stress, strain and youngs modulus.

2 A wire of 16 mm dia. is subjected to a tensile load of 2000 kg. Find the stress and strain if young's modulus E = 2 x 10^{16} kg/cm^2.

3 A wire is of 2 metres long and its area of cross section is 0.78 mm^2. If 78 kg weight is suspended on this wire, then the length of the wire is increased by 1.4 mm. Find out stress, strain and youngs modulus of elasticity.

4 A wire 2800 mm long is stretched by 0.5 mm, when a weight of 9 kg is hung on it, its diameter is 2 mm. Calculate stress and youngs modulus for the substance of the wire.

5 A force of 1000 kg is applied axially on rod of 12 mm diameter the original length is 100 mm. If modulus of elasticity is 2 x 10^{12} kg/cm^2. Calculate the stress and strain developed in the rod.

6 A steel wire 3.2 mm diameter and 3.65 metre long stretches by 2.03 mm under the load of 115 kg. Calculate the stress and youngs modulus of elasticity.

7 A mass of 10 kg is hung from a vertical wire 300.25 cm long and 0.0005 sq. cm cross section. When the load is removed the wire is found to be 300 cm long. Find the modulus of elasticity for the wire material.

Ultimate stress and Working stress

The minimium load at which a material develops failure is called as ultimate load or breaking load. The stress produced in a material at ultimate load is called as ultimate stress or breaking stress.

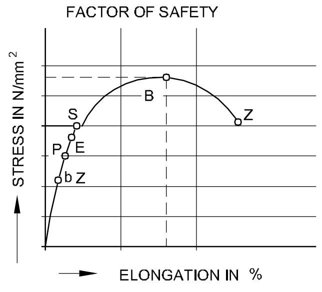
$$\therefore \quad \text{Ultimate stress} = \frac{\text{Ultimate load}}{\text{Area of original cross section}}$$

The load which is considered safe for the machine element is known as safe load or working load and the corresponding stress at this load is called as safe stress or working stress.

$$\therefore \quad \text{Safe stress} = \frac{\text{Safe load}}{\text{Area of original cross section}}$$

Factor of safety (Fig 1)

Fig 1



FACTOR OF SAFETY

The ratio of ultimate stress to working stress (i.e. safe stress) is known as factor of safety. The ratio of ultimate load to the safe load may also be termed as factor of safety. It has no unit. Hence it is expressed in a number.

$$\text{Factory of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

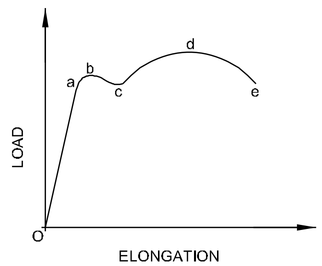
or

$$\text{Factor of safety} = \frac{\text{Ultimate load}}{\text{Safe load}}$$

Stress-Strain graph

Load-extension graph (Fig 2)

Fig 2



CURVE SHOWING RELATIONSHIP BETWEEN LOAD AND ELONGATION

A metal (say mild steel) is subjected to increasing load and the extensions are measured with an extensometer. On plotting a graph between the loads and elongations produced, in the beginning, there is a straight-line relationship. It continues up to 'a' which is called the limit of proportionality, i.e. up to 'a' in Fig 2 'Stress is proportional to strain'.

Point b denotes the elastic limit. Below this point, the body regains its original shape, if the load is removed. Beyond this point the body does not recover its original shape completely, even if the load is removed.

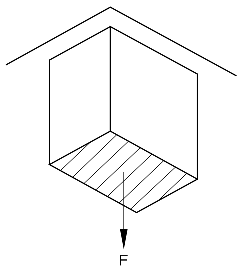
Upto a point beyond the elastic limit, a considerable amount of elongation takes place even with a slight increase in load. The point C where it occurs, is called the yield point.

At 'd' the maximum or the ultimate load is reached. After this, a waist or local contraction is formed in the specimen, and fracture occurs as illustrated in Figure.

Example

A standard steel bar of 30 mm square cross section is subjected to tensile stress. If the factor of safety is 4 and ultimate stress is 370 N/mm² determine the load to which the bar is subjected. (Fig 3)

Fig 3



STRESS-STRAIN

$$\frac{\text{Ultimate stress}}{\text{Working stress}} = \text{Factor of safety} = 4$$

$$\text{Working stress} = \frac{370}{4}\ \text{N/mm}^2$$

Area of cross section = 900 mm²

$(a^2 = 30^2 = 900,\ a = \sqrt{900} = 30)$

Load = Working tensile stress x area

$$= 900\ \text{mm}^2 \times \frac{370}{4}\ \text{N/mm}^2$$

$$= 83250\ \text{N}$$

Example

1 A rod of ϕ60 mm is subjected to a maximum tensile load of 1600 kg. Calculate the stress and strength of the material. If the factory of safety is 5.

Dia. of rod = 60 mm

Tensile load F = 1600 kg

i Stress $= \frac{F}{A} = \frac{1600}{\pi \times 30 \times 30}$

$= 0.5658$ kg/mm^2 Ans.

ii Factor of safety = 5

Factor of safety $= \frac{\text{Ultimate stress}}{\text{Working stress}}$

$5 = \frac{\text{Ultimate stress}}{0.5685 \text{ kg/mm}^2}$

Ultimate stress = 5 × 0.5658 kg/mm²

Strength of the material = 2.829 kg/mm²

2 Find the safe load which can be suspended from a 4.2 mm dia. wire. If the ultimate stress is 25 kg/mm² and the factor of safety is 4.

Dia. of wire d = 4.2 mm

Ultimate stress U.S = 25 kg/mm^2

Factor of safety (FS) = 4

FS. $= \frac{\text{Ultimate stress}}{\text{Working stress}}$

$4 = \frac{25 \text{kg/mm}^2}{\text{W.S}}$

WS $= \frac{25}{4}$ kg/mm^2

$= 6.25$ kg/mm^2

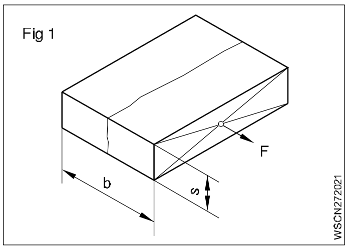
Stress $= \frac{F}{A}$

6.25 kg/mm^2 $= \frac{F \text{kg}}{\pi \times 2.1 \times 2.1 \text{mm}^2}$

F $= 6.25 \times \pi \times 2.1 \times 2.1$ kg

Safe load F = 86.6 kg

Assignment

1 

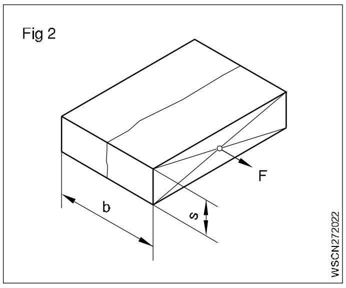
A = 60 x 15 mm^2

Rm = 370 N/mm^2

F = ______ N

Rm = Ultimate stress

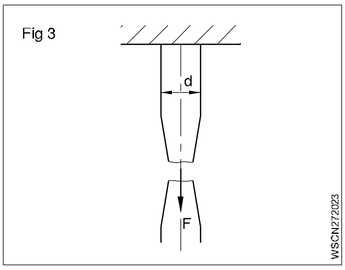
F = Breaking Force

2 

A = 25 x 6 mm^2

F = 63000 N

Rm = ______ N/mm^2

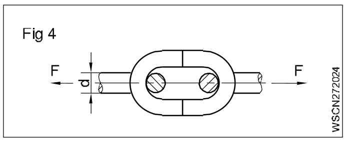
3 

A = 490.87 mm^2

F = 206.22 kN

d = ______ mm

Rm = ______ N/mm^2

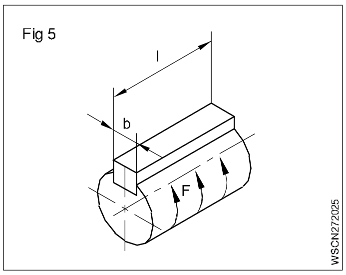
4 

F = 19000 N

Rm = 420 N/mm^2

Factor of safety = 5

d = ______ mm

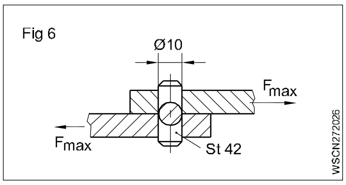
5 

F = 35000 N

Working shear stress = 110 N/mm^2

b = 10 mm

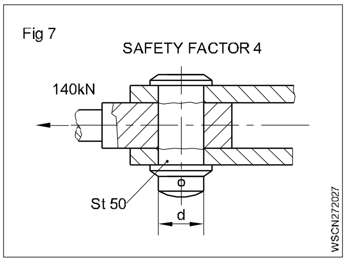
l = ______ mm

6 

Ultimate tensile stress = 420 N/mm^2

d = 10 mm

Shear force Max = ______ N

7 

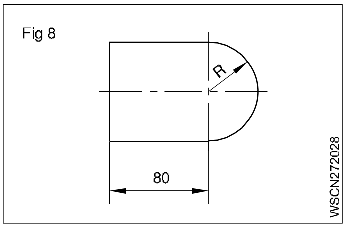
F = 140 kN (Tensile)

Factor of safety = 4

Ultimate tensile stress = 500 N/mm^2

Ultimate shear stress = 400 N/mm^2

d = ______ mm

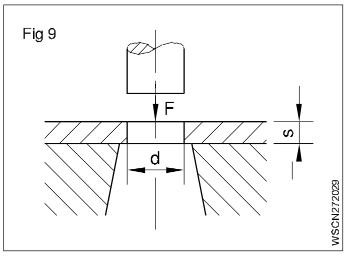
8 

Ultimate tensile strength
= 370 N/mm^2

Shear force
= 380 kN

R = 30 mm

Thickness of plate punched
= ______ mm

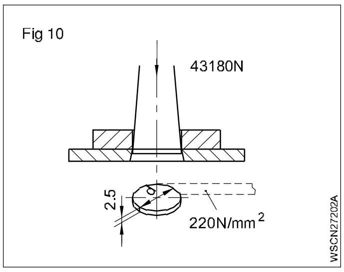
9 

Ultimate tensile strength = 330 N/mm^2

s = 3 mm

F = 60 kN
(Shear force)

d = ______ mm

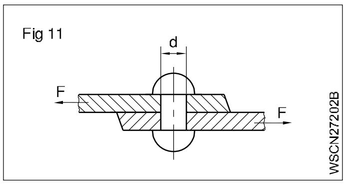
10 

F = 43180 N

s = 2.5 mm

Ultimate tensile strength
= 220 N/mm^2

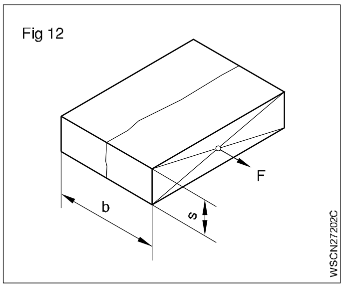
d = ______ mm

11 

d = 12 mm

F = 36.2 kN

Ultimate tensile strength = ____ N/mm^2

12 

Tensile load
= 15000 N

Ultimate tensile strength
= 9.5 N/mm^2

s = 20 mm

Width `b' = ______ mm

13 A steel rod whose diameter is 1 cm and 60 cm in length. This rod is pulled at both ends by a force of 700 kg. If modulus of elasticity of steel is 2.1×10^6 kg/cm^2, find out increase in length of rod and strain produced in it.

14 A steel rod 1.5 metres long and of 30 mm diameter is pulled at both ends by a force of 1500 kg. If modulus of elasticity of steel is 2.4×10^6 kg/cm^2, determine increase in length of rod and strain produced in it.

15 A steel rod of 1.5 cm diameter and 8 metres long pulled by a forced of 80 kg at both ends. Find out the expansion and strain on the rod. The coefficient of elasticityE=2.10×10^6 kg/cm^2.

16 Find out the load which can be suspended from a 3.2 mm dia wire taking the factor of safety as 2. Ultimate stress is 25 kg/mm^2.

17 A rod of 60 mm dia is subjected to a maximum tensile load of 1600kg. Calculate the stress and strength of the material if the factor of safety is 5.

18 A wire of length 3.5 m and diameter 0.35 mm is stretched by a force of 2kg weight. If the elongation is 4 mm. Calculate the young's modulus of the material of wire.

19 A mass of 1kg is suspended from a metal wire 100 cm long and 0.5 mm diameter. An increase in length of wire equal to 2 mm is observed. Calculate the young's modulus of wire.

20 A 4 metre long copper wire of diameter 3 mm is used to support a mass of 50kg. What will be the elongation of the wire. Young's modulus of elasticity for copper is 7×10^{10} N/mm^2.

21 Calculate the change in length of a rod of dia 16 mm and 160 mm long when it carries a load of 40KN. Take E = 200000 N/mm^2.

22 A hollow C.I. column with a wall thickness of 2 cm is subjected to an axial compressive load of 80 tonnes. If the maximum stress is not to be exceeded 1 tonne per cm^2, determine the internal diameter of column. Calculate compressive strain, if E = 950 tonnes per cm^2.